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The role of particle collisions in pneumatic transport

By M. Y. LOUGE[1], E. MASTORAKOS[1] AND J. T. JENKINS[2]
[1] Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, USA
[2] Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, NY 14853, USA

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We analyse the dilute, steady, fully developed flow of relatively massive particles in a turbulent gas in the context of a vertical pipe. The idea is that the exchange of momentum in collisions between the grains and between the grains and the wall plays a significant role in the balance of forces in the particle phase. Consequently, the particle phase is considered to be a dilute system of colliding grains, in which the velocity fluctuations are produced by collisions rather than by the gas turbulence. The balance equations for rapid granular flow are modified to incorporate the drag force from the gas, and boundary conditions, based on collisional exchanges of momentum and energy at the wall, are employed. The turbulence of the gas is treated using a one-equation closure. A numerical solution of the resulting governing equations provides velocity and turbulent energy profiles in agreement with the measurements of Tsuji *et al.* (1984).

1. Introduction

Gas–solid flows satisfy the principles of mass, momentum and energy balance. While the Navier–Stokes equations govern the motion of the gas phase, there is not yet universal agreement upon the way of incorporating the particle phase. One approach is to treat this phase as a continuum and use some form of averaging to obtain the balance laws. Then, depending on the situation, the fluxes and sources of momentum and energy for the particle phase are modelled in order to close the system of equations.

For small particles, dilute in a turbulent gas, momentum transfer in the particle phase is due to turbulent diffusion of the particles. In this regime, Elghobashi & Abou-Arab (1983) have derived a two-fluid k-ϵ closure that relates the Reynolds stress of the particles to gradients of the mean particle velocity using an eddy viscosity that is a fraction of the eddy viscosity of the gas. This analysis predicts the reduction of turbulent energy observed in the presence of small particles (e.g. Modaress, Tan & Elghobashi 1984). Other works in this regime include those of Pourahmadi & Humphrey (1983), Chen & Wood (1985) and Berker & Tulig (1986). However, these analyses do not apply to flows with massive particles.

For massive particles, the experiments of Soo, Ihrig & El Kouh (1960) show that the intensity of the particle velocity fluctuations may exceed that of the fluid, an observation that cannot be explained if the particles respond only to the turbulence. Min (1967) attributes this high particle 'turbulence' to particle–wall collisions. Lourenco, Riethmuller & Essers (1983) also focus attention on collisions when predicting profiles of density and gas and particle velocities in a 10 cm wide horizontal duct loaded with 500 μm glass particles. By analogy with molecular dynamics, these authors introduce a particle velocity distribution function that, they assume, is determined as a solution of a Boltzmann transport equation by particle collisions, rather than by the gas turbulence. In other words, they assume that the gas influences the mean velocity of the particles, but that it has no effect on their random motion. In their work, this assumption is justified, because the ratio of particle relaxation time to a typical large-eddy turbulent timescale is large (Hinze 1972). Sinclair & Jackson (1989) have recently presented a model for vertical gas–solid flow in a pipe that also treats the particle phase as a rapid granular flow. They produce a surprising variety of flow regimes in the context of the model. However, they ignore the gas turbulence and, consequently, do not distinguish the regimes in which collisions dominate the interactions of particles with the turbulent eddies. Neither do they attempt a quantitative comparison of their results with experimental data.

Here, a more detailed analysis of dilute gas–solid flow in a vertical pipe is undertaken. Particles are assumed to be sufficiently massive to be unaffected by the turbulent velocity fluctuations of the gas. The particle phase is treated as a rapidly flowing granular material in which momentum and energy are transferred by the velocity fluctuations of the particles. The momentum balance for rapid granular flow given, for example, by Jenkins & Savage (1983) is modified to include a drag term that provides the force necessary to suspend the particles and the corresponding energy balance includes rates of production and dissipation associated with interactions between the particles and the gas. Finally, the turbulence of the gas phase is treated using a one-equation closure.

Here, we focus on fully developed, steady flows. This assumption is crucial to the analysis for it permits us to carry out averages for both the gas and particle properties on long vertical strips. For the gas, we equate these with ensemble averages or with the time averages determined in the experiments. For the particles, we assume that they are equivalent to averages based on a velocity distribution function. Then, no matter how dilute the flow, the control volumes can always include a large enough number of particles to define the appropriate averages, so the dispersed particles may be regarded as a continuum.

We obtain numerical solutions of the resulting balance laws, constitutive relations and boundary conditions and compare these with the experimental profiles of particle velocity, gas velocity, and turbulent gas fluctuations reported by Tsuji, Morikawa & Shiomi (1984). The agreement of this data with the results of the analysis suggests that particle collisions play an important role in flows involving relatively massive particles, even when the particle phase is rather dilute.

2. Hydrodynamics

In this section, we outline the hydrodynamics for dilute, fully developed, steady flow in a vertical pipe of relatively massive particles of uniform diameter d. We focus on particles massive enough to be unaffected by the velocity fluctuations in the turbulent gas. In other words, the hydrodynamic relaxation time of the particle velocity fluctuations is much greater than a typical roll-over time of the turbulent eddies that is based on their integral lengthscale l and their root-mean-square (r.m.s.) turbulent velocity u'.

2.1. *Particle phase*

In the regime of interest, particles are suspended by the drag force exerted by the mean gas flow, but their velocity fluctuations are the result of collisions with other particles or with the wall. For the particle phase we adopt the assumption of molecular chaos central to the treatment of collisions in rapid granular flows (Jenkins & Savage 1983) and we assume that interparticle collisions are nearly elastic and frictionless. For the conditions of Tsuji *et al.* (1984), the product of the collision frequency and the hydrodynamic relaxation time of the velocity fluctuations varies between 3 and 230. Consequently, a particle loses only a small fraction of its fluctuation energy in and between collisions, so the velocity distribution function for the particles is nearly Maxwellian.

2.1.1. *Momentum*

With these assumptions, well-established results from the kinetic theory of gases lead to constitutive relations for the pressure and shear stress in the particle phase (Chapman & Cowling 1970). The resulting momentum equation for the rapidly flowing particles is then modified to incorporate the drag force from the gas and the gravitational force:

$$\rho_{\mathrm{p}}\frac{\partial[(1-\epsilon)\,v_i]}{\partial t}+\rho_{\mathrm{p}}\frac{[(1-\epsilon)\,v_i v_j]}{\partial x_j}=\frac{\partial S_{ij}}{\partial x_j}+\frac{\rho_{\mathrm{p}}}{T}(1-\epsilon)\,(u_i-v_i)-(1-\epsilon)\,g_i\rho_{\mathrm{p}}, \tag{1}$$

where u_i and v_i are the average gas and particle velocities, p is the gas pressure, g_i is the gravitational acceleration, ϵ is the voidage, ρ_{p} is the material density of the particles, T is the hydrodynamic relaxation time of the mean relative velocity (slip velocity) between the phases, and

$$S_{ij}=-N\delta_{ij}+\frac{5\pi^{\frac{1}{2}}}{96}\rho_{\mathrm{p}}\,d\Theta^{\frac{1}{2}}\left[\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right] \tag{2}$$

is the total stress transmitted through the particle phase. The particle pressure N is related to the particle volume fraction $(1-\epsilon)$ by an expression analogous to an equation of state in a molecular gas:

$$N=\rho_{\mathrm{p}}(1-\epsilon)\,\Theta, \tag{3}$$

where Θ is the 'granular temperature', expressed in terms of the r.m.s. particle velocity fluctuations v' as $\frac{3}{2}\Theta=\frac{1}{2}v'^2$. In these dilute flows, terms in the momentum exchange between the phases that are proportional to the gradient of the voidage are neglected.

For fully developed, axisymmetric flow, the momentum balance in the vertical direction reduces to

$$0=\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rS)+\frac{\rho_{\mathrm{p}}}{T}(1-\epsilon)\,(u-v)-\rho_{\mathrm{p}}(1-\epsilon)\,g, \tag{4}$$

where u and v are the vertical components of the mean gas and particle velocities, r is the radial coordinate and S, the particle shear stress on surfaces at constant radius, is given through (2) by

$$S=\frac{5\pi^{\frac{1}{2}}}{96}\rho_{\mathrm{p}}\,d\Theta^{\frac{1}{2}}\frac{\mathrm{d}v}{\mathrm{d}r}. \tag{5}$$

In this simple flow, the particle momentum balance in the radial direction becomes

$$\mathrm{d}N/\mathrm{d}r = 0, \tag{6}$$

so that N is constant across the pipe.

For the very dilute flow of relatively large particles, the length λ of the mean free path between collisions may be comparable to the radius R of the pipe. In this case, the transport of particle momentum is similar to that in the free-molecule regime of gas dynamics and, by analogy with the heuristic treatment of shear stress in Couette flow there (e.g. Vincenti & Kruger 1977), so we adopt the following correction for S:

$$S = \frac{5\pi^{\frac{1}{2}}}{96}\rho_\mathrm{p} d\Theta^{\frac{1}{2}}\frac{\mathrm{d}v}{\mathrm{d}r}\frac{1}{1+\lambda/R}, \tag{7}$$

where $\lambda = d/[6\sqrt{2}(1-\epsilon)]$. Unlike (5), this expression for shear stress approaches zero as $(1-\epsilon)$ vanishes, so that (4) remains valid in this limit.

The hydrodynamic relaxation time T of a particle is defined in terms of the drag force on one particle by

$$\frac{\rho_\mathrm{p}}{T} = C_d|u-v|\frac{3\rho}{4d}, \tag{8}$$

where ρ is the gas density. For the drag coefficient C_d we adopt the empirical expression

$$C_d = \frac{24}{Re_\mathrm{p}}(1+0.15Re_\mathrm{p}^{0.687}), \tag{9}$$

which is valid in the range $0 < Re_\mathrm{p} < 800$, where $Re_\mathrm{p} = |u-v|\rho d/\mu$ is the Reynolds number based on the average slip velocity between the phases and μ is the viscosity of the gas (Clift, Grace & Weber 1978).

2.1.2. *Energy*

In order to close the equations describing the particle phase, an energy balance is written for the determination of the granular temperature Θ. Following Jenkins & Savage (1983), it is

$$\tfrac{3}{2}\rho_\mathrm{p}(1-\epsilon)\frac{\partial\Theta}{\partial t}+\tfrac{3}{2}\rho_\mathrm{p}(1-\epsilon)\,v_i\frac{\partial\Theta}{\partial x_i} = -\frac{\partial q_i}{\partial x_i}+S_{ij}\frac{\partial v_i}{\partial x_j}-D_1-D_2, \tag{10}$$

where q_i is the diffusive flux of particle fluctuation energy, given by kinetic theory as

$$q_i = -\frac{25\pi^{\frac{1}{2}}}{128}\rho_\mathrm{p} d\Theta^{\frac{1}{2}}\frac{\partial\Theta}{\partial x_i},$$

and D_1 and D_2 are rates of dissipation per unit volume due, respectively, to the inelasticity of the particles and to their interaction with the gas.

The second term of the right-hand side of (10) is the working of the stress through the mean particle velocity gradient. In the dilute limit, D_1 is given for nearly elastic particles as

$$D_1 = \frac{24(1-e)\,\rho_\mathrm{p}}{\pi^{\frac{1}{2}}d}\Theta^{\frac{3}{2}}(1-\epsilon)^2, \tag{11}$$

where e is the coefficient of restitution for a particle–particle collision (Jenkins & Savage 1983). The expression for D_1 is obtained by considering the kinetic energy lost in each collision, then averaging over all possible collisions.

The rate of energy dissipation per unit volume D_2 results from the working of the fluctuating force exerted by the gas through the fluctuating velocity of the particles. This fluctuating force includes a drag component associated with the slip velocity and a Basset component that arises from the rapid variation of the slip velocity with time. A calculation of the Basset component indicates that in the experiments of Tsuji *et al.* (1984) it is at most 20% of the drag component. Upon ignoring its contribution, D_2 may be written as

$$D_2 = -(\rho_p/T)(1-\epsilon)\overline{v_i'(u_i'-v_i')} = -(\rho_p/T)(1-\epsilon)(\overline{u_i'v_i'}-3\Theta), \tag{12}$$

where primes indicate fluctuating velocities and the overbar denotes the average.

The term $\overline{u_i'v_i'}$ is the correlation between the velocity fluctuations of the gas and those of the particles. Koch (1990) has recently calculated this term for a dilute gas–solid suspension at very low particle Reynolds number in the limit where solid-body collisions determine the particle velocity distribution function. He finds that

$$\overline{u_i'v_i'} = \frac{4}{\pi^{\frac{1}{2}}}\frac{d(u_i-v_i)(u_i-v_i)}{T_0\,\Theta^{\frac{1}{2}}}, \tag{13}$$

where T_0 is the Stokes relaxation time. In the experiments of Tsuji *et al.* (1984), the Reynolds number based on the r.m.s. fluctuating slip velocity lies well above one. In this regime there are no theoretical prediction for $\overline{u_i'v_i'}$. In this case we adopt the functional form given by (13) but use the relaxation time T_1 given by the analogue of (8) based on the r.m.s. fluctuating slip velocity between the two phases. This extension of Koch's expression is analogous to the extension of the Stokes drag coefficient to other than low particle Reynolds number. In the situations considered here its contribution is typically dominated by other terms.

For fully developed, axisymmetric flow, (10) becomes

$$0 = -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rq)+S\frac{\mathrm{d}v}{\mathrm{d}r}-D_1-D_2, \tag{14}$$

where
$$q = -\frac{25\pi^{\frac{1}{2}}}{128}\rho_p\, d\Theta^{\frac{1}{2}}\frac{\mathrm{d}\Theta}{\mathrm{d}r}.$$

To treat the very dilute flow of large particles, we modify this expression using a correction similar to that in (7):

$$q = -\frac{25\pi^{\frac{1}{2}}}{128}\rho_p\, d\Theta^{\frac{1}{2}}\frac{\mathrm{d}\Theta}{\mathrm{d}r}\frac{1}{1+\lambda/R}.$$

2.1.3. *Boundary conditions*

Boundary conditions for the particle phase have been calculated by Jenkins (1991) who considers collisions of inelastic frictional spheres with a frictional wall. For relatively small values of the coefficient of sliding friction μ_f, the resulting tangential momentum balance and the balance of energy are, respectively,

$$S = -\mu_f N \tag{15a}$$

and
$$q = -\tfrac{3}{8}N(3\Theta)^{\frac{1}{2}}[\tfrac{7}{2}(1+e_w)\mu_f^2-(1-e_w)], \tag{15b}$$

where e_w is the coefficient of restitution for a particle colliding with the wall. These relationships have been derived based on the assumption that the coefficient of friction is so small that the point of contact of a particle always slips during a collision.

Finally, because the flow is axisymmetric, the boundary conditions at the centreline of the pipe are that S and q vanish there.

2.2. *Gas phase*

2.2.1. *Momentum*

In our treatment of the fully developed, steady particle flow, we considered a control volume of infinitesimal width but arbitrary extent in the vertical direction and assumed that the velocity distribution function depended only upon the radial position of the control volume. Further, we restricted attention to particles that are too massive to be affected by the turbulent velocity fluctuations; their velocity fluctuations are governed by a velocity distribution function $f(\boldsymbol{v}, \boldsymbol{x}, t)$. The particle volume fraction $(1-\epsilon)$ is related to this velocity distribution function by

$$\iiint f(\boldsymbol{v}, \boldsymbol{x}, t)\,\mathrm{d}\boldsymbol{v} \equiv (1-\epsilon)/(\tfrac{1}{6}\pi)\,d^3.$$

In the flow under consideration f and, consequently, ϵ are independent of time, so $\epsilon' = 0$. Thus, a Reynolds decomposition of the momentum equation leads to

$$\rho\frac{\partial(\epsilon u_i)}{\partial t}+\rho\frac{\partial(\epsilon u_i u_j)}{\partial x_j} = -\frac{\partial(\epsilon p)}{\partial x_i}+\frac{\partial(\epsilon \tau_{ij})}{\partial x_j}-\frac{\rho_p}{T}(1-\epsilon)(u_i-v_i) \tag{16}$$

and

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\tfrac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}-\rho\overline{u_i' u_j'}, \tag{17}$$

respectively.

We adopt a closure of the Reynolds stress based on the eddy viscosity μ_t:

$$\rho\overline{u_i' u_j'} = -\mu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\tfrac{2}{3}\delta_{ij}\left(\rho k+\mu_t\frac{\partial u_k}{\partial x_k}\right), \tag{18}$$

where $k = \frac{1}{2}\overline{u_k' u_k'}$ is the turbulent kinetic energy per unit mass of the gas. The eddy viscosity is assumed to be given by the one-equation closure described, for example, by Reynolds (1976):

$$\mu_t = C_\mu \rho k^{\frac{1}{2}} l, \tag{19}$$

where l is the turbulent mixing length and $C_\mu = 0.49$. Following common practice in pipe flow, we assume that near the wall the mixing length is proportional to the distance from the wall and that near the centre of the pipe it is constant:

$$\frac{l}{R} = \begin{cases} \kappa(1-r/R), & r/R \geqslant 0.7; \\ 0.3\kappa, & r/R \leqslant 0.7, \end{cases} \tag{20}$$

where $\kappa = 0.41$ is von Kármán's constant.

For axisymmetric, steady, fully developed flow, the vertical component of (16) becomes

$$0 = -\frac{\partial(\epsilon p)}{\partial z}-\frac{\rho_p}{T}(1-\epsilon)(u-v)+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\epsilon\tau_{rz}), \tag{21}$$

where $\tau_{rz} = (\mu+\mu_t)\,\mathrm{d}u/\mathrm{d}r$ and z is the upward vertical coordinate. The corresponding radial component is

$$0 = -\frac{\partial(\epsilon p)}{\partial r}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\epsilon\tau_{rr})-\frac{\epsilon\tau_{\theta\theta}}{r},$$

where $\tau_{rr} = -\rho\overline{u_r'^2}$ and $\tau_{\theta\theta} = -\rho\overline{u_\theta'^2}$. Using (18), we find that

$$\rho\overline{u_r'^2} = \rho\overline{u_\theta'^2} = \tfrac{2}{3}\rho k.$$

Consequently, the radial component is

$$\frac{\partial(\epsilon p)}{\partial r} = \tfrac{2}{3}\rho\frac{\mathrm{d}(\epsilon k)}{\mathrm{d}r}. \tag{22}$$

We do not require a solution to (22); from it, it follows that $\partial(\epsilon p)/\partial z$ is a constant independent of r and z in fully developed flow.

2.2.2. *Kinetic energy*

The balance of turbulent kinetic energy is obtained by multiplying the instantaneous gas momentum equation by u_i', taking the time average of all terms, and subtracting the energy equation for the mean flow. For steady flows involving massive particles the resulting equation reduces to eleven terms:

$$\begin{aligned}\frac{\partial(\rho\epsilon u_j k)}{\partial x_j} &= -\overline{u_i'u_j'}\frac{\partial(\rho\epsilon u_i)}{\partial x_j}-\frac{\partial}{\partial x_i}(\epsilon\overline{p'u_i'})-\overline{p'u_i'}\frac{\partial\epsilon}{\partial x_i}-\rho\epsilon\,\overline{u_i'u_j'\frac{\partial u_i'}{\partial x_j}}\\ &\quad-\rho\epsilon u_i\overline{u_i'\frac{\partial u_j'}{\partial x_j}}+\frac{\rho_p}{T}(1-\epsilon)\,\overline{u_i'(v_i'-u_i')}+\mu\epsilon\overline{u_i'\frac{\partial}{\partial x_j}\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}\\ &\quad+\mu\frac{\partial\epsilon}{\partial x_j}\overline{u_i'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}-\tfrac{2}{3}\mu\epsilon\overline{u_i'\frac{\partial^2u_j'}{\partial x_i\partial x_j}}-\tfrac{2}{3}\mu\frac{\partial\epsilon}{\partial x_i}\overline{u_i'\frac{\partial u_j'}{\partial x_j}}.\end{aligned} \tag{23}$$

We adopt the closures proposed by Elghobashi & Abou-Arab (1983) for the velocity and pressure correlations. In particular, we use (18) for the Reynolds stress and a gradient diffusion model for the turbulent transport of k:

$$\rho\epsilon\overline{u_j'\frac{u_i'u_i'}{2}} = \epsilon\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}, \tag{24}$$

where σ_k is a turbulent Prandtl number appropriate for the diffusion of k. Following common practice we assume that $\sigma_k = 1$. We ignore a counter-gradient diffusive flux.

In fully developed, axisymmetric flow, (23) simplifies to

$$\begin{aligned}0 &= \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left\{r\epsilon\left(\frac{\mu_t}{\sigma_k}+\mu\right)\frac{\mathrm{d}k}{\mathrm{d}r}\right\}+\mu_t\epsilon\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2-\rho\epsilon E-\frac{\rho_p}{T}(1-\epsilon)\,\overline{u_i'(u_i'-v_i')}\\ &\quad-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(\epsilon r\overline{p'u_r'})-\overline{p'u_r'}\frac{\mathrm{d}\epsilon}{\mathrm{d}r}-\tfrac{2}{9}\mu\frac{k}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\epsilon}{\mathrm{d}r}\right)+\tfrac{4}{9}\mu\frac{\mathrm{d}\epsilon}{\mathrm{d}r}\frac{\mathrm{d}k}{\mathrm{d}r}+\tfrac{10}{9}\mu\frac{k}{\epsilon}\left(\frac{\mathrm{d}\epsilon}{\mathrm{d}r}\right)^2,\end{aligned} \tag{25}$$

where $E = (\mu/\rho)\,\overline{\partial u_i'\partial u_i'/\partial x_j\partial x_j}$ is the isotropic turbulent dissipation rate.

The first term of (25) represents the turbulent and viscous diffusion of k. The second term is the rate of production of k by the working of the mean shear. Because the particles modify the mean gas velocity profile through the drag term in (21), they affect the gas velocity gradient and, through the second term in (25), the production of k. The third term is the isotropic dissipation of k. For dilute conditions, it is closed as described by Reynolds (1976): $E = C_2 k^{\frac{3}{2}}/l$, where $C_2 = C_\mu^3$. The fifth and sixth terms are the work of turbulent pressure fluctuations. The next three terms are viscous dissipation terms. According to Hinze (1975), the relative pressure fluctuations p'/p are at least an order of magnitude smaller than the relative velocity fluctuations u'/u. Because the fifth term is much less than order $\epsilon\rho u'^3/R$, which is itself smaller than the turbulent diffusion term of order $\epsilon\rho u'^3/l$, it is negligible. Because the last four terms involve the average voidage gradient $\partial\epsilon/\partial r$, which is of order $(1-\epsilon)/R$, in dilute flow they may be ignored. With these simplifications, (25) becomes

$$0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left\{r\epsilon\left(\frac{\mu_t}{\sigma_k}+\mu\right)\frac{\mathrm{d}k}{\mathrm{d}r}\right\}+\mu_t\epsilon\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2-\rho\epsilon E-\frac{\rho_p}{T}(1-\epsilon)(2k-\overline{u_i'v_i'}). \tag{26}$$

Our experience in analysing the experiments of Tsuji *et al.* (1984) is that the production of turbulent kinetic energy is dominated by the working of the mean shear, and that its dissipation is primarily governed by the isotropic dissipation rate E.

2.2.3. *Boundary conditions*

Because the closure outlined above is not valid in the viscous and buffer layers, boundary conditions for the gas phase at the wall are enforced at a dimensionless distance $y^+ = \rho(R-r)u^*/\mu \approx 30$ from the wall. The parameter $u^* = (\epsilon\tau_0/\rho)^{\frac{1}{2}}$ is the shear velocity, where τ_0 is the gas shear stress at the wall. In the dilute flow, we assume that the gas velocity profile near the wall is not greatly affected by the presence of the particles. In this case, the gas velocity at small values of y^+ is given by the universal 'law of the wall':

$$\frac{u}{u^*} = \begin{cases} 5\ln y^+ - 3, & 5 \leqslant y^+ \leqslant 30; \\ (1/\kappa)\ln y^+ + 5.7, & y^+ \geqslant 30. \end{cases} \tag{27}$$

The shear velocity is calculated from a global momentum balance obtained by adding the momentum balances of the gas and particle phases and integrating between $r = 0$ and $r = R$:

$$\partial(\epsilon p)/\partial z = -2\rho u^{*2}/R + 2S_0/R - g\rho_p(1-\bar{\epsilon}), \tag{28}$$

where S_0 is the particle shear stress evaluated at the wall and $\bar{\epsilon}$ is the average voidage across the pipe.

For the turbulent kinetic energy, we take the boundary condition at the wall to be

$$\partial k/\partial r = 0. \tag{29}$$

Provided that the molecular viscosity μ is much smaller than the eddy viscosity μ_t (a condition typically met for $y^+ \approx 30$), this condition is equivalent to equating the production and dissipation of turbulent kinetic energy. In particle-laden flows, unlike pure gas flows, equating the production and the dissipation of k at the boundary does not yield a straightforward algebraic expression for k because of the last term of (26). In this context, (29) is as exact and more convenient than specifying a value for k at the wall in terms of u^{*2}.

The system of equations (21) and (26) was first solved for pure gas ($\epsilon = 1$) subject to the boundary conditions (27)–(29) at the wall and the requirement that the radial

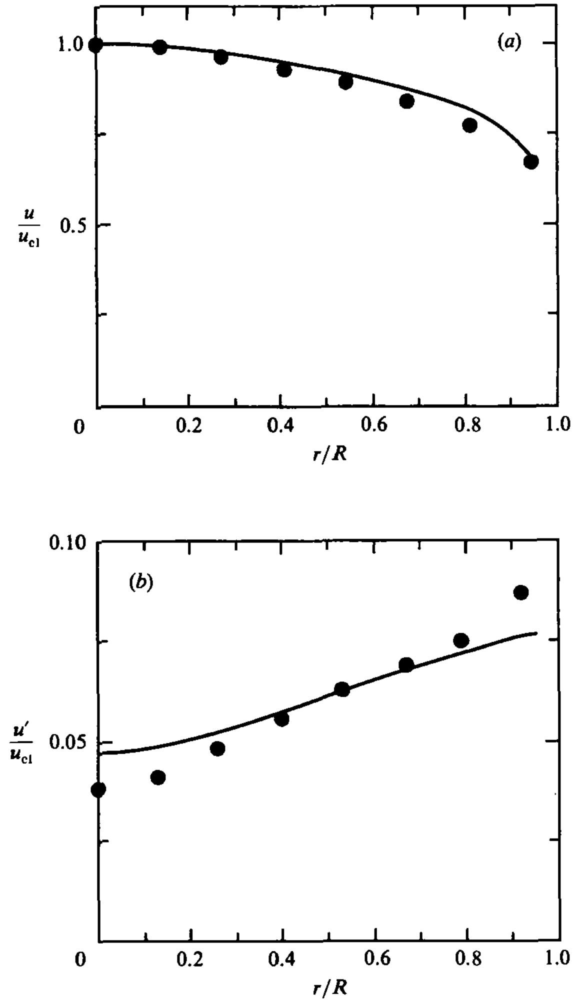


FIGURE 1. Calculated profiles of (*a*) gas velocity u and (*b*) r.m.s. gas velocity fluctuations u' for clear gas, normalized by the centreline gas velocity $u_{cl} = 13.4$ m/s. The solid circles represent the data of Tsuji *et al.* (1984).

derivative of u and k vanish at the centreline. As indicated in figure 1, the predicted profiles of gas velocity and turbulent kinetic energy are in satisfactory agreement with the measurements of Tsuji *et al.* (1984) despite the rather rudimentary turbulence closure used. In addition, the predictions of the skin friction coefficient at the wall, $f \equiv 2\tau_0/\rho\bar{u}^2$, are within 10% of the Blasius correlation for a smooth pipe, $f = 0.079Re^{-\frac{1}{4}}$, where Re is based on the pipe diameter and the average gas velocity $\bar{u}$ over the cross-section.

3. Results and discussion

In this section, we compare the detailed measurements of Tsuji *et al.* (1984) with the predictions of our analysis. Using a laser-Doppler anemometer, these authors measured the profiles of mean gas velocity, mean particle velocity, and gas velocity fluctuations in a vertical pipe of 30.5 mm diameter under fully developed, steady

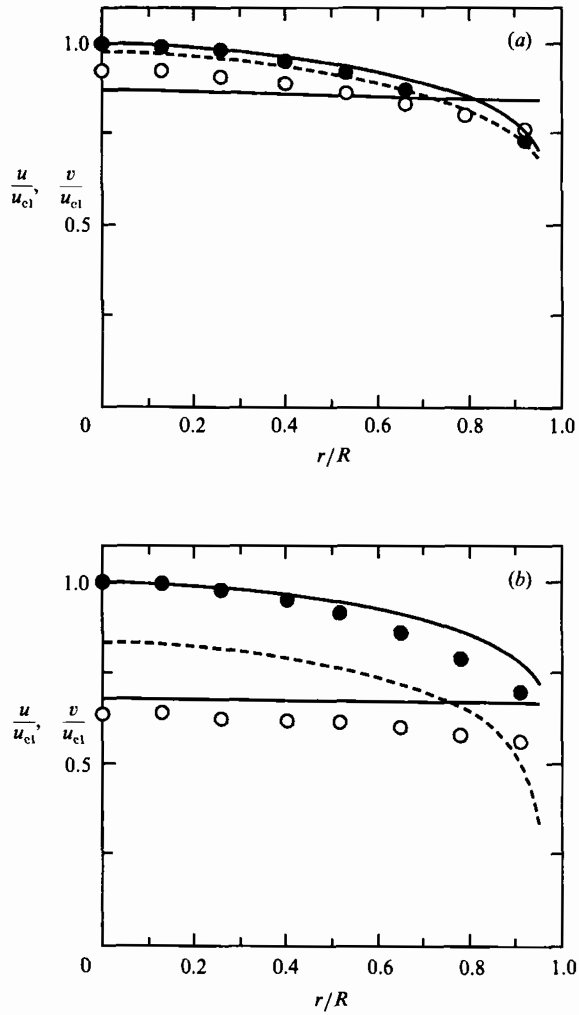


FIGURE 2. Calculated profiles of normalized gas velocity u/u_{cl} and particle velocity v/u_{cl} for relatively dilute flows of (*a*) 200 μm and (*b*) 500 μm particles. The solid and open circles represent the data of Tsuji *et al.* (1984) for gas and particle velocities, respectively. The dashed lines represent particle velocities predicted by an analysis that would ignore particle shear. The conditions are: (*a*) $u_{cl} = 18.9$ m/s, $m = 1.0$; and (*b*) $u_{cl} = 9.65$ m/s, $m = 1.1$.

flow. In these experiments, polystyrene spheres with density $\rho_p = 1020$ kg/m^3 and diameters in the range 200 μm to 3 mm were suspended in air. Ratios of particle-to-gas mass flow rates m (loading) were as high as 3.6. For these conditions, the hydrodynamic relaxation time of the particles is between 60 ms and 850 ms, while the slowest turbulent timescale is of order $R/u \sim 2$ ms. Clearly, the particles cannot follow the gas turbulence.

The system of ordinary differential equations (3), (4), (6), (14), (21), (26) and boundary conditions constitutes a nonlinear, coupled, two-point boundary-value problem. We solve this system numerically using the quasi-linearization method of Bellman & Kalaba (1965). In the calculation we assume a coefficient of restitution $e = 0.9$ for particle–particle collisions, $e_w = 0.7$ for particle–wall collisions, and a coefficient of dynamic friction $\mu_f = 0.2$ between a particle and the wall. The

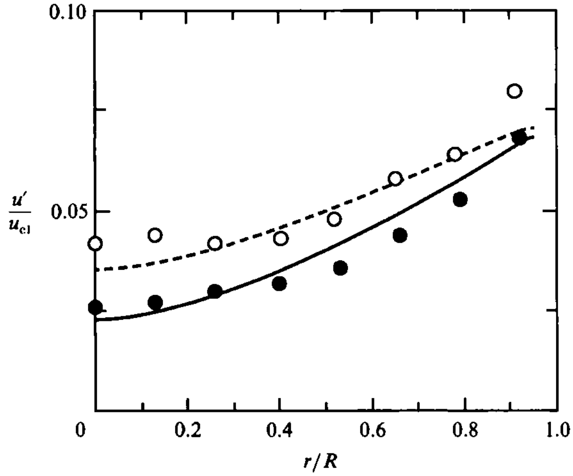


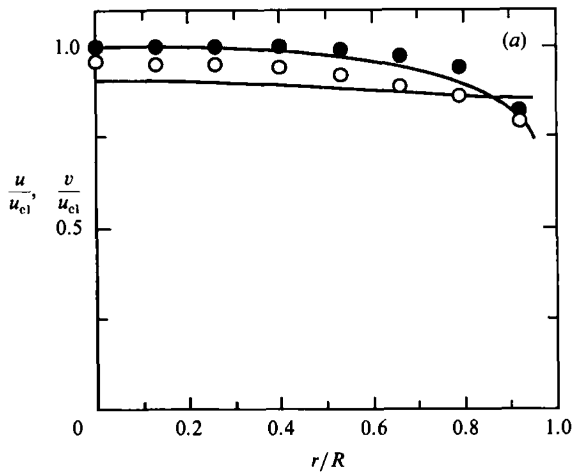
FIGURE 3. Calculated profiles of normalized r.m.s. gas velocity fluctuations u'/u_{cl} for relatively dilute flows. The solid circles represent the data of Tsuji *et al.* (1984) for 200 μm particles, $u_{cl} =$ 12.8 m/s, $m = 1.3$; the open circles are 500 μm particles, $u_{cl} = 13.3$ m/s, $m = 1.3$. The solid and dashed lines are the respective predictions of the analysis. We predict $(1-\bar{\epsilon}) = 0.16\%$ and $\Theta^{\frac{1}{2}}/u_{cl} \approx 1.8\%$ for 200 μm particles and $(1-\bar{\epsilon}) = 0.18\%$ and $\Theta^{\frac{1}{2}}/u_{cl} \approx 2\%$ for 500 μm.

coefficients of restitution are consistent with the observations of Govan, Hewitt & Ngan (1989), who filmed the trajectories of glass spheres transported in a small pipe under conditions similar to those of Tsuji *et al.* (1984). For particle–wall collisions, these authors estimated that $0.52 < e_w < 0.95$.

In particle-laden flows, two parameters determine the operating conditions, i.e. gas velocity and solid loading. Rather than specifying these values at the onset of the computations, it was more convenient to input to the program the overall pressure gradient and the particle volume fraction at the wall. These were iterated until the gas velocity at the centreline and the loading were within 1 % of the measured values.

Figures 2 to 5 compare the measurements of Tsuji *et al.* and the predictions of our analysis for a range of particle diameters and loadings. Note that the predicted particle velocity at the wall is positive. Comparable observations were made by Lee & Durst (1982) in a similar situation. At first sight, these observations might be surprising. Because the gas velocity is zero at the wall, one might expect the particles to fall. In fact, there is a region near the wall where the particles can acquire a velocity higher than that of the gas. This effect is due to the shear stress in the particle phase. Particles further from the wall are lifted by the gas and, through collisions, transfer momentum to particles closer to the wall. In figures 2 and 5(*a*), the dashed line represents the prediction of an analysis that ignores this shear stress in the balance of forces for the particle phase. Such a treatment clearly fails to reproduce the observed particle velocity profile. Therefore, stresses in the particle phase are essential for an accurate description of the flow. In addition, because the loading is proportional to the average product of the particle velocity and volume fraction across the pipe, the agreement of the calculated particle velocity with the measured values suggests that equations (3) and (6) adequately predict the particle volume fraction, at least with respect to the mean.

Figure 3 compares the measured and predicted r.m.s. velocity fluctuations for 200 μm and 500 μm particles at relatively low particle loadings. The present analysis predicts correctly the magnitude of the r.m.s. velocity fluctuations and the trend



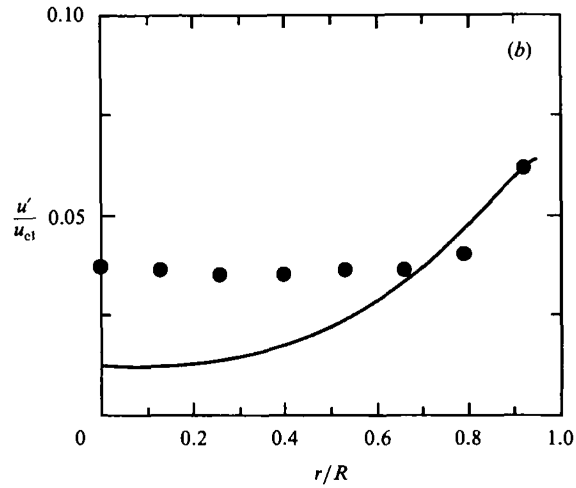
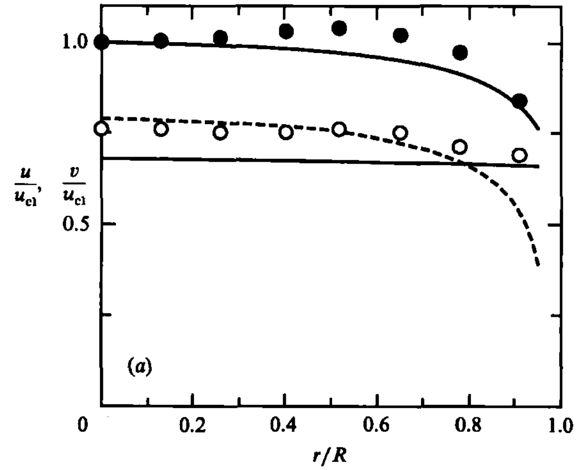


FIGURE 4. Calculated profiles of (*a*) u/u_{cl}, v/u_{cl}, and (*b*) u'/u_{cl} for relatively dense suspensions of 200 μm particles. In (*a*), the solid circles represent the data of Tsuji *et al.* (1984) for u/u_{cl} and the open circles are the data for v/u_{cl}. In (*b*), the solid circles are the data for u'/u_{cl}. The solid lines are the prediction of the analysis. The conditions are: (*a*) u_{cl} = 14.6 m/s, m = 4.2; (*b*) u_{cl} = 10.8 m/s, m = 3.2. For (*b*), we predict $(1-\bar{\epsilon}) = 0.43\%$ and $\Theta^{\frac{1}{2}}/u_{cl} \approx 1.8\%$.

towards larger fluctuations for larger particles. Figures 4 and 5 concern the largest particle loadings considered by Tsuji *et al.* (1984) for 200 μm and 500 μm particles. Here, the mean velocities are well predicted by the analysis, but the r.m.s. velocities are underpredicted near the centreline of the pipe. In this case it may be desirable to refine the turbulence closure of equations (18)–(20) to obtain a better agreement with the measured gas velocity fluctuations at higher particle loadings.

Figure 6 compares our predictions of the total pressure gradient $-\partial(\epsilon p)/\partial z$ with the measurements of Tsuji *et al.* (1984). Despite the crudeness of the turbulence closure, the predictions of equation (28) are within 15% of the measured values. For these conditions, the contribution of the particle weight $g\rho_p(1-\bar{\epsilon})$ to the total pressure gradient is relatively small ($\approx$20%), and that of the particle shear stress $-2S/R$ is even smaller ($\approx$8%).



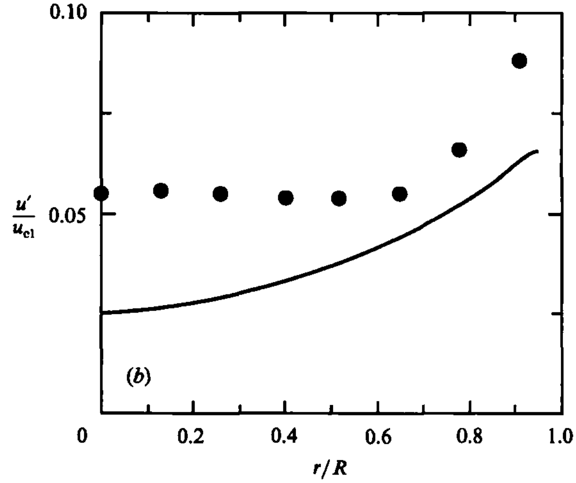


FIGURE 5. Calculated profiles of (*a*) u/u_{cl}, v/u_{cl}, and (*b*) u'/u_{cl} for relatively dense suspensions of 500 μm particles. Symbols have the same meaning as in figure 4. The conditions are: (*a*) $u_{cl} = 8.07$ m/s, $m = 3.6$; and (*b*) $u_{cl} = 10.7$ m/s, $m = 3.4$. For (*b*), we predict $(1-\bar{\epsilon}) = 0.5\%$ and $\Theta^{\frac{1}{2}}/u_{cl} \approx 2.2\%$.

Unfortunately, our predictions of the profiles of granular temperature Θ cannot be compared with the experiments of Tsuji *et al.* (1984), who did not measure the particle velocity fluctuations or the concentration of the particle phase. Such measurements would add considerably to the understanding of this flow regime. In particular, they would help clarify the form of the particle velocity distribution function, as well as the nature of the correlation $\overline{u_i' v_i'}$. If such experiments are undertaken, we recommend that the coefficients of restitution e and e_w and the coefficient of sliding friction μ_f be reported, because these values affect the magnitude of the granular temperature Θ.

Our experience in analysing the experiments of Tsuji *et al.* (1984) is that the mean velocity profiles and the gas velocity fluctuations are rather insensitive to the actual values of the coefficients of restitution. In contrast, because the coefficient of friction governs the magnitude of the particle shear at the wall and the flux of granular

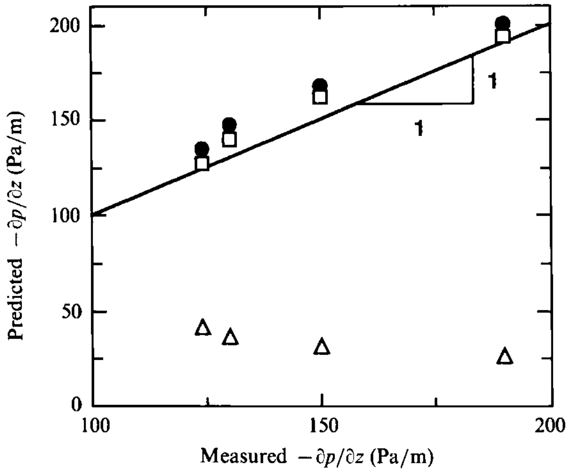


FIGURE 6. Calculated versus measured fully developed pressure gradient $-\partial(\epsilon p)/\partial z \approx -\partial p/\partial z$ for 500 μm particles. The circles are the total pressure gradient predicted by equation (28). The triangles represent the predicted contribution of the particle weight. The squares are the sum of the contributions of the gas shear and the particle weight. The total particle mass flow rate is 0.03 kg/s. The mean gas velocities are, from left to right, $\bar{u} = 11.6$, 13.0, 14.8, and 17.2 m/s. For these conditions, we predict, respectively, $(1-\bar{\epsilon}) = 0.39$, 0.35, 0.30, and 0.25% and $\Theta^{\frac{1}{2}}/u_{cl} \approx 1.8$, 1.7, 1.6, and 1.4%.

energy, it has a strong influence on the flow over a range of particle diameters. For example, for typical conditions (500 μm polystyrene particles, $\bar{u} = 10$ m/s, $m = 1$), the total predicted pressure gradient rises sharply above $\mu_f \approx 0.33$. These observations suggest that the boundary conditions of the particle phase play an important role in the flow.

Finally, our analysis predicts the behaviour of the vertical pressure gradient observed in pneumatic transport lines. At constant loading, the predicted fully developed pressure gradient first decreases with gas velocity. Here the gas shear $2\rho u^{*2}/R$ is the dominant term in equation (28). However, as the gas velocity decreases, the particle volume fraction must increase to maintain the constant loading, so that the weight of the particle phase becomes increasingly important. Eventually, the particle weight dominates equation (28) and the total pressure gradient increases with decreasing gas velocity. Eventually the suspension can no longer be sustained and the 'choking' occurs. It is interesting to note that, for the dilute flows under consideration, the particle shear predicted by the analysis plays a relatively minor role in the total pressure gradient, despite its importance in the predictions of the particle mean velocity profiles.

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